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Abstract

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This paper develops a structural model of demand for a network good to provide a rigorous definition of critical mass. Using simulations, we demonstrate that our model of critical mass can be operationalized easily and can generate theoretically grounded insights about critical mass phenomena identified in empirical settings. We then propose a number of extensions to illustrate the flexibility of our model.

Keywords: critical mass, network effects, diffusion of innovations, compatibility

JEL Classification: C53, L14, M37

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> "the explanation [of the traditional perspective] for the S-shaped curve of adoption sounds much like critical mass (…) [yet] the notion of critical mass calls for important modification of diffusion theory in the particular case of interactive innovations"
>
> (Rogers, 1995, p. 324)

1. Introduction

When PayPal went public in 2002, what set it apart from the many failures in the dot.com market was that it was considered to have reached *critical mass*.[1] When Joltage attempted to build a WiFi hotspot network, providers of complementary products were hesitant to invest before the technology had reached *critical mass*.[2] The popular press carries an abundance of similar references to the all-important critical mass point at which a technology's success is virtually guaranteed. The main difficulty observers face however is that critical mass is an ad-hoc phenomenon – you know it when you see it.

The marketing literature defines critical mass as "the point after which further diffusion becomes self-sustaining." (Rogers, 2003, p. 343). However, many other definitions of critical mass exist.[3] Many of these involve setting an ad-hoc threshold on empirically observable measures—market penetration or sales growth rates—rather than a rigorous theoretical underpinning. Conversely, existing theory-driven definitions are not always helpful because they are not easily operationalized in an empirical setting. We close this gap by offering a simple and empirically tractable definition of critical mass based on a structural demand model with network effects with the aim of developing an econometrically feasible way of identifying and estimating critical mass.

The model developed in this paper emphasizes the role of network effects in an S-shaped adoption curve. The link between network effects and diffusion is well established in the literature (Rohlfs,

[1] The Economist, *Party like it's 1999?* 23 February 2002, p. 65.
[2] The Economist, *Making Wi-Fi pay*. 6 June 2002, p. 53.

1974; Granovetter, 1978; Markus, 1987, Cabral, 1990, 2006; Loch and Huberman, 1999), but to our knowledge it has not been used so far to identify critical mass empirically. As shown in this literature, network effects can generate multiple stable equilibria separated by an unstable point referred to as critical mass. That is, diffusion takes off when a switch between a low-adoption and a high-adoption equilibrium occurs at a certain point of the diffusion process. Using this logic, our model offers some new insights into the critical mass phenomenon. Most importantly, we show that critical mass is a function of the price paid by the adopters rather than an absolute number of market penetration or past sales. That is, our model predicts that the critical mass of adopters is smaller (larger) when the price is lower (higher).

For a certain functional specification we are able to derive a Bass-type diffusion equation (Bass, 1969). The estimated parameters in the Bass model and in our model, however, have different interpretations. The primitives of our model recovered from the estimated parameters refer to the strength of network effects rather than the probability of being "infected", as Bass (1969) does not consider network effects.

We derive our main results based on a basic version of the model, which makes a number of simplifying assumptions. Specifically, we focus on the case of full compatibility and non-durable goods. These assumptions are relaxed and alternative functional specifications proposed later, when we introduce some extensions to allow for a wider scope of applications.

In the following section, we review the existing literature on critical mass and introduce a structural model of network demand that lets us recover the strength of network effects as estimated parameters in Section 3. We then illustrate how these parameters can be used to estimate and identify critical mass by simulating a specific functional form in Section 4. In Section 5, we introduce extensions of the basic model including alternative functional forms, durable goods, imperfectly compatible technologies, and switching costs. Section 6 concludes.

[3] Other closely related concepts used in the literature include catastrophe point, take-off, and punctuated equilibrium.

2. Related Literature

There are several strands of literature related to critical mass. The first stream of research derives theoretical or conceptual explanations as to why certain markets display critical mass phenomena. Markus (1987) outlines a set of qualitative indicators to identify interactive media markets with critical mass behavior and finds that the underlying production function and consumer heterogeneity are especially important in such markets. Oren et al. (1982) and Dhebar and Oren (1985) propose models of dynamic pricing in network markets and find that "boundary pricing" (charging high-preference consumers close to their willingness to pay) may lead to premature "stalling" of the network at less than full market penetration. Conversely, if prices are sufficiently low early on, the market displays critical mass, i.e. spontaneous and rapid growth of adopters. Loch and Huberman (1999) simulate the diffusion of a new technology with network effects and find that rapid (i.e. critical mass-like) transition from an old to a new standard can occur if consumers have a high rate of experimentation and the new technology improves rapidly. Economides and Himmelberg (1995) study different market structures and their propensity to generate critical mass-like phenomena. However, their definition of critical mass differs from others in that they define it as the minimum sustainable network size, i.e. the smallest stable equilibrium. They find that perfectly competitive markets will usually display critical mass, while monopolistic markets need not. Finally, Cabral (1990, 2006) proposes a model similar in spirit to ours and finds that the equilibrium adoption path may display discontinuities, i.e. critical mass phenomena. In Cabral (1990, 2006), these "catastrophe points" occur only if network effects are sufficiently strong. Common to all the papers discussed above is that they attempt to outline a number of circumstances (e.g. demand and network parameters, production technologies, market structure etc.) under which critical mass-like phenomena can occur in an industry. While most papers give anecdotal evidence of market dynamics consistent with their models, none of them offer a robust and flexible model of critical mass which readily lends itself to estimation.

The second stream of research focuses on finding a heuristic for discovering and predicting critical mass in real data. Here, the common definition of critical mass is a percentage of overall market potential (Mahler and Rogers, 1999, Cool et al., 1995), while other papers use the concept of sales takeoff (Golder and Tellis, 1997, Agarwal and Bayus, 2002) which is closely related to critical mass since once takeoff occurs, the product (or technology) is assumed to reach full saturation eventually. The threshold level for critical mass varies anywhere between 10% (Mahler and Rogers, 1999) and 25% (Cool et al., 1995), and the takeoff point is defined as a sales growth rate in relation to the installed base of previous sales (Golder and Tellis, 1997). Related to these papers are the diffusion models pioneered by Bass (1969) and the categorization of adopters (e.g., Rogers, 2003, Mahajan et al., 1990)). The transition from a product appealing predominantly to pioneers and early adopters to one adopted by the mass market is what qualitatively constitutes critical mass (Rogers, 2003). In interactive innovations (e.g. network technologies), this takeoff point may be reached earlier (Mahler and Rogers, 1999). What all these papers have in common is that they assume critical mass phenomena as given, and identify parameters to predict its timing (e.g. Golder and Tellis, 1997) and magnitude (e.g. Cool et al., 1995). The source of critical mass, however, is not typically specified via a theoretical model of demand.

In this paper, we combine both streams and propose a model of demand that can display critical mass phenomena under certain conditions while offering a simple but flexible way of estimating market parameters to give a simple heuristic of estimating critical mass phenomena and characteristics of demand. In a number of extensions, we show that our basic model can be extended to accommodate empirical settings from a number of network industries.

3. Basic Model

3.1 Willingness to Pay and Network Effects

Our definition and operationalization of critical mass is based on a demand model with network effects.[4] In each instance of time, consumers decide whether to buy the good under consideration or not depending on their net benefit. In our basic model the good is assumed to be non-durable.[5] Examples include subscription to a payment system like a credit card, or a communication service like email or

[4] Our basic model draws on Cabral (1990).
[5] Section 5.2 presents an extension to the durable good case.

mobile phone. Most importantly, the good displays network effects, so that the installed base of adopters (subscribers) affects consumer willingness to pay for the good (service).

Each consumer's willingness to pay for the subscription is influenced by her type and the number of existing subscribers. The set of subscribers is referred to as network. We assume that there is a measure one of infinitely-lived consumers with unit demand for the subscription. Consumer *v's* preferences are represented by the willingness-to-pay function $u(v, x(t-\delta))$, where v is the individual preference parameter, $x(t-\delta)$ is lagged network size at time t, and the perception lag δ a non-negative number.[6] We further assume that the individual preference parameter v is distributed over the interval [0,1] according to a CDF $F(v)$, and that $u(v, x(t-\delta))$ is strictly increasing and continuous in v. The parameter v thus establishes a rank ordering of consumers in their willingness to pay. We assume that the ranking is invariant with respect to changes in $x(t-\delta)$.

We introduce lagged network size $x(t-\delta)$ to the willingness-to-pay function for two main reasons. First, it captures network effects in demand for the good. Second, the perception lag δ is an equilibrium selection device which yields a unique diffusion path of the network.[7] However, it also raises questions about consumers' rationality – why would a consumer base purchasing decisions on events that occurred some time in the past? There are two justifications for this: First, small perception lags will result in near-rational behavior, so we could approximate rationality with our model. Indeed, Cabral (1990) shows that if δ is infinitely small, consumers are rational because their subscription decisions are identical to the ones made by forward-looking consumers. Second, we believe that realistically consumers will not have access to the current number of users of a good, but rather on previously published figures – which resembles a perception lag. We therefore believe that our assumption of a perception lag is both realistic and necessary to derive an empirically tractable strategy to identify critical mass. While it would be preferable to have the actual expectations of consumers, this data is

[6] We expand on the role of δ in Section 3.4.
[7] As shown in Cabral (1990), when δ=0 there are infinitely many equilibrium diffusion paths. A positive δ implies that consumers cannot coordinate their subscription decisions leading to a unique equilibrium diffusion path. An alternative approach followed in Economides and Himmelberg (1995) is to allow consumers to coordinate in order to reach the critical mass.

rarely available. Working with lags then represents a sensible alternative, although the approximation of rational consumers' expectations through lags depends on the frequency of our observations.[8]

3.2 Short-Run and Long-Run Subscription Demand

At time t, consumer v decides whether to subscribe to maximize her net utility from participation in the considered market:

(1) $$u(v, x(t-\delta)) - p(t).$$

We assume that there is one market price *p(t)*. If (1) is non-negative, the consumer will join the network, otherwise she will stay out.[9]

Denote by v_t^* the type of the consumer who is indifferent between joining the network or staying out at time t. The indifferent type is determined by the following equation:

(2) $$u(v_t^*, x(t-\delta)) = p_i(t).$$

Since the type v establishes a rank ordering of consumers in their willingness to pay, all consumers with $v \geq v_t^*$ will subscribe. Define

(3) $$H(v_t^*) \equiv 1 - F(v_t^*),$$

so $H_i(\cdot)$ equals the number of consumers subscribed to the network at time t. The state equation describing network size at time t —the short-run demand—is then given by:

(4) $$x(t) = H(v_t^*).$$

In steady state no consumer can increase her utility by subscribing or unsubscribing. The network thus stays constant over time, which gives rise to the following long-run demand condition:

(5) $$x(t) = x(t-\delta).$$

[8] Clearly, the less frequent the observations, the worse the approximation.
[9] This "static" decision rule in our dynamic model is appropriate for non-durable goods and will be relaxed in 5.2.

Long-run demand is reached when the market is fully saturated and there are no more consumers to further fuel diffusion. However, this need not be the case. Long-run demand can also be short of full saturation depending on prices and consumer preferences. This is an important feature of our model which differentiates it from the Bass (1969) model in which full saturation is always reached in the long run. In other words, our model can also accommodate failed products.

It is also worth noting that the steady-state equilibrium in our model coincides with the static fulfilled-expectations equilibrium, which is routinely used in the economics literature (see, e.g. Rohlfs, 1974 and Katz and Shapiro, 1985).

3.4 Network Dynamics: Critical Mass and Diffusion Take-Off

In this section, we assume that the CDF of $F(v)$ and the willingness-to-pay function $u(v, x(t-\delta))$ are continuously differentiable in all arguments. Related to our model, Cabral (1990) analyzes the equilibrium network size path of a network technology and shows that for sufficiently strong network effects and lag length δ approaching zero the equilibrium adoption path is unique and discontinuous.

The equilibrium adoption path is described by equation (4). Since $H(\cdot)$ maps the change in network size from time $t-\delta$ to t, it is convenient to think of it as of a function of the lagged network size $x(t-\delta)$. To see how network externalities and price affect diffusion, we calculate the derivatives of $H(\cdot)$ with respect to lagged network size $x(t-\delta)$ and price p in the Appendix. It turns out that the slope of $H(\cdot)$ increases in the extent of network effects measured by $\eta \equiv \frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)}$. Lemma 1 in the Appendix formalizes the link between the extent of network effects and the slope of $H(\cdot)$, which in turn determines diffusion dynamics.

Figure 1 illustrates these dynamics. In the top panel we draw $H(\cdot)$ as a function of lagged network size $x(t-\delta)$. Given the steady-state condition (5), the long-run equilibria of the network size coincide with the fixed points of $H(\cdot)$. As shown in the Appendix, without network effects the function $H(\cdot)$ is a horizontal line and cannot have more than one fixed point. A combination of positive network effects and a bell-shaped distribution of types v is likely to result in a function $H(\cdot)$ with multiple long-run equilibria similar to Figure 1. The dynamics in our model then let us discriminate among these multiple steady-states. Suppose market price is p^* in Figure 1. According to state equation (4) then, network size will evolve as indicated in the top panel. If it starts at some size $x < x_1$ it will eventually reach x_0, if $x > x_1$ it will end up in x_2. If $x = x_1$, it will stay there, but any arbitrarily small shock will lead to an equilibrium at x_0 or x_2. Therefore, x_0 and x_2 are stable steady states, while x_1 is unstable.

INSERT FIGURE 1 ABOUT HERE

Figure 1: Stable vs. Unstable Equilibria

We can apply the same logic to any price p. Lemma 2 in the Appendix states that lowering the price shifts the function $H(\cdot)$ upwards (although not necessarily in a parallel shift). Drawing the steady states for each price gives the long-run demand $D(p)$ in the lower panel of Figure 1. This yields our definition of a critical mass point:

PROPOSITION 1. *Downward-sloping parts of the long-run demand $D(p)$ consist of stable equilibria, while the upward-sloping parts are unstable, i.e. consist of critical-mass points. For unstable equilibria to exist, network effects must be sufficiently strong.*

The intuition of Proposition 1 is that downward-sloping parts of the demand correspondence are *locally* “well-behaved” – every price *p* has a single corresponding long-run network size given by the demand $D(p)$. Conversely, critical mass points are unstable in the sense that they divide regions of attraction towards the stable equilibria. Once the installed base of subscribers reaches critical mass there is a qualitative change in the diffusion process: The switch from low-adoption to high-adoption equilibrium occurs and diffusion takes off.

Now consider the case when there is no initial installed base and the price falls over time. That is, let $p(t)$ be a continuous and decreasing function of time and let $p(0) > p_h$ (as in Figure 1) and $x(p(0))$ be the unique steady-state network size given $p(0)$. As price falls, network size initially follows the low-adoption steady-state. Eventually the price reaches p_l and immediately after, network size jumps to the high-adoption steady-state and grows further along it. Formally, this diffusion pattern is correct for infinitely small δ (Cabral, 1990). If the perception lag is strictly positive, consumers are myopic with respect to network size – they do not recognize that the network is going to grow in the current period. As a consequence, equilibrium network size does not follow exactly but rather tends towards the steady-state equilibrium. Instead of the discontinuous jump in network diffusion, a rapid take-off occurs and diffusion takes an S-shape.

This dynamic perspective helps us understand the equilibrium selection rule implicitly assumed in our model by the lag structure as we do not rely on coordination among consumers to obtain a discontinuous jump.[10] Instead, rapid diffusion emerges spontaneously at price p_l. For take-off to occur at a price higher than p_l, the supplier needs to move the process, e.g. through temporary discounts, free sampling, etc. to help grow the installed base to the critical mass level. Our model now lets us formulate some predictions about the comparative static behavior of the critical mass point:

PROPOSITION 2. *If sufficiently strong network effects exist to generate multiple steady-state equilibria, critical mass is reached at lower (higher) installed base for lower (higher) price. Ceteris paribus, stronger network effects imply critical mass at lower installed base and/or higher price.*

The first part of Proposition 2 follows immediately from Proposition 1: Because the unstable equilibria and therefore critical mass points are positioned on the upward sloping part of the long-run demand function, a higher price implies higher critical mass and vice versa. The second part of Proposi-

[10] Note that it would be Pareto-optimal to jump to the larger steady-state network size before price falls below p_l. However, this would require the coordination of consumers' subscription decisions to reach critical mass.

tion 2 is also intuitive: With stronger network effects it takes a smaller installed base to swing a consumer with a given intrinsic valuation if we keep the price and distribution of types unchanged.

3.4 Optimal Pricing along the Diffusion Path

We do not impose any structure on the supply side of the good since the main focus of our paper is on identifying conditions under which demand for a good displays critical mass phenomena. Moreover, from an econometric perspective we do not require any structure for the supply relation to be able to correctly estimate the network effect parameter and identify critical mass, as endogeneity issues regarding the price variable can be resolved with instrumental variable techniques.

4. Empirical Specification

4.1 An Example of a Functional Specification

Our next step towards operationalizing critical mass is to specify functional forms for the underlying theoretical model. The specification in this section has been chosen for two reasons. First, this specification yields the diffusion equation as a simple linear (in parameters) equation, which is convenient to work with empirically. Second, this diffusion equation nests the seminal Bass (1969) model. However, it would be straightforward to use other specifications instead.[11]

We specify consumer *v*'s willingness-to-pay function as follows:

(6) $$u\left(v, x_{i,t-1}\right) = v + cx_{i,t-1} + dx_{i,t-1}^2,$$

where *c* and *d* are parameters determining the extent of network effects,[12] with the square term capturing possible nonlinearities, e.g. diminishing marginal network effects usually assumed in the theoretical literature (Swann, 2002). Moreover, assume *v* to be uniformly distributed over $(-\infty, a]$ with density

[11] An example of this is given in 5.1.
[12] Note that in the empirical model δ will be determined by data frequency. Consequently, we replace δ with 1 meaning "one period" from now.

$b > 0$. For convenience, population size is not normalized to one as in the theoretical model. With the distribution of types specified above, the population is infinite to avoid corner solutions.[13]

Given these functional forms, diffusion equation (4) becomes

(7) $$x_t = ab - bp_t + bcx_{t-1} + bdx_{t-1}^2 .$$

The structural parameters of this model can be recovered from the following estimation equation:

(8) $$x_t = \alpha + \beta p_t + \gamma_1 x_{t-1} + \gamma_2 x_{t-1}^2 + \varepsilon_t ,$$

where ε_t denotes an iid error term. Equation (8) simplifies to the original Bass model if $\beta = 0$ (i.e. price does not matter for network diffusion).

The structural parameters of the model can be identified from the coefficients in (8). Simple algebra yields the highest consumer type in the population $a = -\alpha/\beta$ and the density of the distribution of types $b = -\beta$. The parameter α can be interpreted as the number of consumers with positive valuation of the mobile telephone service given zero network size. The network effects parameters c and d are $-\gamma_1/\beta$ and $-\gamma_2/\beta$, respectively.

The interpretation of the identified structural network effects parameters c and d directly is difficult for two reasons. First, these parameters do not have natural boundaries and are not unit free, thus troubling any significance thresholds and comparisons across markets. Second, the estimates of network effects parameters are generally not independent of the consumer perception lag δ imposed by the data frequency, further complicating interpretations.[14] Instead, simulation of critical mass and steady state of the diffusion process offer a more intuitive and practical way to interpret the estimated model.

[13] Alternatively, the distribution support could be bounded from below to limit the population of consumers and the bound assumed to be low enough in order to avoid the necessity of considering corner solutions, when all consumers subscribe.

[14] The perception lag and the strength of network effects are substitutes in the sense that shorter perception lags and stronger network effects both accelerate diffusion and lead to critical mass being reached earlier.

4.3 Simulations

The parameters estimated in our simple model can now be used to identify combinations of installed base ($x_{i,t-1}$) and prices ($p_{i,t}$) that give an upward-sloping long-run demand curve, i.e. points at which critical mass occurs. In this section, we show how critical mass depends on the parameters in our model, especially network effects and the price sensitivity of demand. In our basic model, the number of firms does not affect critical mass directly as all consumers are part of a common network. However, firms competing on a common network are likely to charge Bertrand prices, which in turn affects critical mass.

4.3.1 Increase in Network Effects

The effect of an increase in network effects is shown in Figure 2. The other model parameters are set as a = 100 (maximum willingness to pay for subscription when network size is zero) and b = 0.02 (density of the distribution of consumer types). In the left panel, we can see that with network effects becoming stronger (i.e. increasing c) the demand function becomes more concave and eventually features an upward-sloping part (critical mass). In the right panel, we find that an increase in d,[15] i.e. a less pronounced effect of decreasing marginal network effects, generates a very similar picture. In accordance with Proposition 2, we observe critical mass (for a given price p) earlier when network effects get stronger (i.e. increasing c or d). This effect is more pronounced in the left panel of Figure 2, as c affects the extent of network effects more than d for small network sizes. In contrast, d dominates the extent of network effects for large network sizes. In particular, a strongly negative d parameter can lead to negative network effects for large networks, for example through congestion effects.

INSERT FIGURE 2 ABOUT HERE

Figure 2: Simulations for Different Network Effects Parameters

4.3.2 Decrease in Price Sensitivity of Demand and Consumer Stand-Alone Valuation

We now show the effects of changes in the distribution of consumer types. Absent network effects, the demand function in our example is just a linear function $x_t = ab - bp_t$. Figure 3 shows the effects of changes in these parameters for an extent network effects set at *c*=100 and *d*=-5. In the left panel, we see that for larger values of *a*, the critical mass point is reached more easily. This is intuitive since *a* determines the stand-alone value of the product. In particular, when the stand-alone value is zero, we have a pure network product, which exemplifies the chicken-and-egg paradox: No consumer wants to subscribe unless there is some installed base of subscribers. In the right panel, we can see the impact of increased density of consumer types' distribution *b*, which determines short-run price sensitivity of demand. It very much resembles the impact of increased network effects. With increased *b* the number of consumers willing to subscribe at each price is higher. At some point, when *b* is high enough, network effects can "kick in", leading to critical mass for relatively high prices.

INSERT FIGURE 3 ABOUT HERE

Figure 3: Simulations for Different Price Sensitivity and Market Size Parameters

In our simulations, we show that in otherwise identical markets (i.e. similar structural parameters and price sensitivity), markets with more pronounced network effects can display critical mass phenomena, while others with more moderate network effects may not. Further, the curvature of the network effect function plays an important role in the likelihood of critical mass being achieved. For example, networks exhibiting congestion effects (i.e. large and negative *d*) would feature critical mass for a much narrower range of prices. Regarding the price sensitivity of demand, more price-sensitive goods are more likely to display critical mass phenomena, suggesting that small changes in price may generate even more extreme changes in the demand for the good than anticipated from a static demand curve.

[15] That is, *d* becoming less negative.

Finally, it is important to stress that the possible shape of steady-state demand as simulated in Figures 2 and 3 is heavily influenced by our functional form assumptions. With these particular assumptions, we do not obtain two downward-sloping parts in the demand, as in Figure 1.[16] However, the simulated demand functions in Figures 2 and 3 approximate the more general function in Figure 1, since the vertical axis above the minimum critical mass point (i.e. $x=0$ for sufficiently high p) belongs to long-run demand too. Thus the intuition behind the critical mass as dividing the high and the low demand regions (zero in the simulations above), remains true even for these simplifying functional assumptions.

5. Extensions

5.1 Alternative empirical specifications

One alternative specification of functional forms combines a log-logistic distribution of types with network effects which enter the willingness to pay multiplicatively. Assume the following willingness-to-pay function:

(9) $$u(v, x_{t-1}) = v x_{t-1}^{c},$$

where c measures the extent of network effects, which in now are not additively separable from the consumer type v as in Section 4. Indeed, equation (9) implies that if network effects are positive (i.e. c is positive), higher consumer types enjoy a network of a given size more than low types. This assumption seems plausible in many settings including telecommunications. For instance, if a high type reflects a heavy intended intensity of use, then heavy users may benefit more from an increased network than light ones. Moreover, assume that v is distributed over $[0, +\infty)$ according to the following log-logistic CDF:

(10) $$F(v) = \frac{m}{1 + \exp(-b(\log(v) - a))},$$

[16] The requirement for this is a long tail in the distribution of types, which captures consumers with very high willingness to pay even if no one else subscribes (the innovators).

where *a* and *b* are parameters determining the distribution's shift and slope, respectively, and *m* is a parameter reflecting the market size. Regardless of the way a log-logistic distribution of types is parameterized, it is intuitively appealing because it approximates the distribution of income in most populations.[17]

Given these functional forms, diffusion equation (4) becomes

$$(11) \qquad x_t = \frac{m}{1 + \exp(-ba + b\log(p_t) - bc\log(x_{t-1}))}.$$

The structural parameters *a*, *b*, *c*, and *m* can easily be identified allowing for simulations of the critical mass analogous to the one in Section 4. One interesting feature of equation (11) is that it can replicate an S-shape even without network effects (i.e. c=0). Critical mass can only exist, however, if network effects are strong enough, as stated in Proposition 1.

5.2 Durable goods

Now consider an extension of the basic model to durable goods (Economides and Himmelberg, 1995). Assume that once the durable good is purchased, it yields infinite stream of utility represented by the willingness-to-pay function $u(v, x(t-\delta))$. Each consumer v chooses to purchase the good (to join the network) at time t_v^* to maximize the present value of the good minus the present value of its cost, i.e

$$(12) \qquad \max_t \left(\int_t^\infty e^{-\rho s} u(v, x(s-\delta))ds - e^{-\rho t} p(t) \right),$$

where ρ is the common, instantaneous discount rate and *p(t)* is the price at time *t*. If the optimization problem (12) is concave, it has a unique solution given by:[18]

[17] An alternative approach taken by Economides and Himmelberg (1995) is to assume that the distribution of types mirrors the distribution of income in the population. Then, one can apply a two-step estimation procedure: Estimate the distribution's parameters in the first step and the willingness-to-pay parameters in the second step.
[18] The sufficient condition for the solution to be unique is that the shadow price $\lambda(t)$ is monotonically decreasing over time, i.e. *p(t)* is decreasing and convex.

(13) $$u(v, x(t_v^* - \delta)) = \rho p(t_v^*) - \frac{dp(t_v^*)}{dt} \equiv \lambda(t_v^*) .$$

Condition (13) says that a consumer of type v is indifferent between adopting at time t_v^* or not, because the utility from having the good at this particular instance of time is exactly offset by the cost of capital plus the value of waiting (lower price in the future). All higher types, however, strictly prefer to buy the good (if they have not done so yet). Condition (13) is then analogous to condition (2) for non-durable goods. The subsequent analysis of the durable goods' case essentially follows that of non-durable goods with the shadow price $\lambda(t)$ replacing regular price *p(t)*.

5.3 Imperfect Compatibility.

In the case of imperfect compatibility, we need to distinguish between different brands of the good. In their purchasing decision consumers choose between firms offering these brands. For simplicity, we assume that the good is *ex ante* homogenous, which means that branding has no other meaning than identifying the firm that offers it.[19] With imperfect compatibility, however, brands will matter because they may have differently-sized networks.

Formally, each consumer's willingness to pay for a given brand is influenced by her type and the network size of that brand. Network size depends on the number of own subscribers, the number of subscribers to competing brands, and the degree of compatibility. We denote each brand by $i(i = 1,2,...I)$ and assume that there is a measure one of infinitely-lived consumers with unit demand for the good or service. Consumer *v's* preference for brand i at time t is represented by the willingness-to-pay function $u(v, x_i(t-\delta))$, where v is the individual preference parameter, $x_i(t-\delta)$ is lagged network size of brand i, and δ is the perception lag, as before. We uphold the assumptions about the individual preference parameter v: It is distributed over the interval [0,1] according to a CDF $F(v)$, and $u(v, x_i(t-\delta))$ is strictly increasing and continuous in v. Again, this establishes a rank ordering of consumers in v assumed to be invariant with respect to changes in $x_i(t-\delta)$. Consumers are thus heterogeneous in their valuation of the good, but they do not have an inherent preference for a particular brand. If brands have equal-sized networks, they are perceived as perfect substitutes by consumers.

If brands are incompatible, each makes up its own network so $x_i(t-\delta)=y_i(t-\delta)$, where $y_i(t-\delta)$ stands for cumulative normalized (by overall market size) subscriptions to brand i. In a more general setting with partial compatibility, brand network size is the weighted sum of its own and all other subscribers. Assuming a symmetric degree of compatibility, we can write this as

(14) $$x_i(t-\delta)=y_i(t-\delta)+w\sum_{j\neq i} y_j(t-\delta),$$

where $w\in[0,1]$ measures the degree of compatibility, and $w=1$ and $w=0$ correspond to perfect compatibility and perfect incompatibility, respectively.

At each time t, consumer v joins one of the networks or stays out of the market to maximize her net utility:

(15) $$u(v,x_i(t-\delta))-p_i(t).$$

If (15) is negative for all brands *i*, she will not join any network.

Denote by $v_{i,t}^*=v^*(x_i(t-\delta),p_i(t))$ the type of the indifferent consumer with respect to brand i at time t. This implies that:

(16) $$u(v_{i,t}{}^*,x_i(t-\delta))=p_i(t).$$

Given our assumptions, the brand i for which $v_{i.t}{}^*$ is lowest is the most attractive brand for all subscribers at time t. Define

(17) $$\underline{v}_t{}^*=\min_i\{v_{1,t}{}^*,v_{2,t}{}^*,\ldots,v_{I,t}{}^*\}.$$

[19] By *ex ante* homogeneity we mean that different brands of the good are perceived by the consumers as intrinsically equal. However, the difference in the consumer valuation of the brands is possible *ex post* if they have networks of different size.

All consumers of a higher type than $\underline{v}_t^*$ subscribe to one of the networks. If $v_{i,t}^*$ is equal for some brands, we assume that adopters choose among them with equal probability. We can now define a function $H_i(.)$ that denotes the number of consumers joining network i at time t:

$$(18) \qquad H_i\left(v_t^*\right) \equiv \begin{cases} \dfrac{1-F\left(\underline{v}_t^*\right)}{\underline{I}_t} & if \quad v_{i,t}^* = \underline{v}_t^* \\ 0 & otherwise \end{cases},$$

where $v_t^* = \left(v_{1,t}^*, v_{2,t}^*, ..., v_{I,t}^*\right)$ is a vector of the indifferent types with respect to each brand i at time t, $\underline{I}_t$ is the number of brands for which $v_{i,t}^* = \underline{v}_t^*$ and $F(.)$ is the distribution function of v. The state equations describing the evolution of each brand's sales over time are now given by

$$(19) \qquad y_i(t) = H_i\left(v_t^*\right).$$

The steady state is reached when the networks stabilize for a given set of parameters—most importantly prices—analogously to the perfect compatibility case. This is expressed by:

$$(20) \qquad y_i(t) = y_i(t-\delta).$$

This setup has two notable features. One of them is persistent firm symmetry. At each time t, every active firm has an equal number of subscribers $y_i(t)$, which is in contrast to the observation that real firms' market shares exhibit persistent differences. The other feature is the degree of competition among firms. If one firm marginally undercuts the others it takes the entire market, which results in fierce price competition.[20] Switching costs, which we consider as another extension in the next section, mediate this tendency of network markets to some extent. In other words, market shares become more "sticky" if existing consumers find it costly to switch suppliers (Farrell and Klemperer, 2007).

5.4 Switching Costs

Suppose switching costs are high enough so that, having chosen one particular brand, a consumer would not switch later on, for example because of penalties for canceling their subscription prematurely, as observed in telecommunications markets. In general, under switching costs and imperfectly compatible brands the consumer choice problem becomes quite complex. Expectations about the market become important, as customers stick to their initial choices. For tractability, however, we assume that switching is possible without costs if prices increase. Moreover, we assume that all networks are equally attractive in expectations. Thanks to these assumptions a static consumer decision based solely on current observation of prices and network sizes is also optimal in a dynamic sense.

Switching costs thus change the demand described in (18) in that only unattached consumers can join a network. We rewrite (18) as

(21) $$H_i^{'}\left(v_t^{*}, v_{t-\delta}^{*}\right) \equiv \begin{cases} \dfrac{F(\underline{v}_{t-\delta}^{*}) - F(\underline{v}_t^{*})}{\underline{I}_t} & if \quad v_{i,t}^{*} = \underline{v}_t^{*} \\ 0 & otherwise \end{cases}.$$

Now, $H_i^{'}(\cdot)$ is the number of *new* consumers choosing brand i at time t. The state equations are then given by

(22) $$y_i(t) = H_i^{'}\left(v_t^{*}, v_{t-\delta}^{*}\right) + y_i(t-\delta).$$

In contrast to equation (19), equation (22) exhibits inertia as any installed-base asymmetry from previous periods will be carried over to the current one. Under the assumption of equal attractiveness in expectations, however, such asymmetries cannot persist. The firms may violate the condition $v_{i,t}^{*} = \underline{v}_t^{*}$ only randomly, which means that in the long run these asymmetries will cancel each other out. Still, the presence of switching costs will discourage firms from aggressive price undercutting, as the possible market share gains will be lower. Moreover, switching costs will prevent new entrants from immediately catching up with incumbents, as the basic model would predict. In fact, the installed base that incumbents grew prior to entry will persist and constitute a competitive advantage over entrants. To be equally attractive and stay in the market, entrants will then need to offer better prices or quality on their networks.

[20] This is the “Bertrand Trap” in the literature (see, e.g. Cabral and Villas-Boas, 2005).

Under a stronger version of the equal attractiveness conditions, $v_{i,t}^* = \underline{v}_t^*$ for all *i* and *t*. In this case, substituting (18) and (21) into (22) and rearranging the terms yields[21]

(23) $$y_i(t) = H_i\left(v_t^*\right) + l_i E_t,$$

where the l_i's are brand-specific constants and E_t is an entry indicator function that is zero pre-entry and one post-entry. Equation (23) is identical to equation (19) up to the term $l_i E_t$, which stands for the installed base advantage of incumbents over entrants, illustrating how entry can persistently break symmetry in our model.[22]

6. Conclusion

In our paper, we develop a simple model of a new technology with network effects to identify possible critical mass phenomena. While most existing papers either focus on the theoretical possibility of critical mass phenomena or an empirical heuristic of diffusion takeoff or critical mass, we combine these two strands and propose a simple theoretical model which can easily be implemented empirically. In line with existing theoretical literature, we identify critical mass points as combinations of price and installed base leading to multiple equilibria. Empirical work has emphasized the role of prices, network effects and installed base on critical mass, a result which we replicate in our theoretical model. Using simulations, we show that the parameters recovered from the empirical implementation of our model can be used to identify and analyze critical mass in a market. In a series of extensions, we illustrate the flexibility of our model to accommodate different distributions of consumer preferences, durable goods, imperfect compatibility and switching cost, all of which are common features of emerging network industries.

Our aim was to build a bridge between theoretical rigor and empirical feasibility. Despite the limitations inevitably imposed, e.g. on strategies available to firms, by our simple model, we believe that it

[21] See the Appendix for details. For simplicity, we consider single entry only. It is straightforward to extend the model to account for multiple entries.

is flexible enough to enable scholars and practitioners to apply it to a variety of empirical settings. In summary, we believe that our model offers a practical alternative to ad-hoc specifications or difficult to implement theoretical models and hope that scholars and practitioners will be able to build on our framework in future research.

[22] Grajek (2007) estimates a model similar to (23) based on data from a mobile telecom market. His focus is on network effects and compatibility among competing mobile networks rather than critical mass though.

Appendix

A.1. Derivatives of the function *H(.)* with respect to *x(t – δ)* and *p*

For simplicity, we slightly abuse the notation in this section by treating price as a constant parameter p. Recall that v_t* is an implicit function of $x(t-\delta)$ and p defined by

(A.1) $u(v_t^*, x(t-\delta)) = p.$

To calculate the derivative of H with respect to the lagged network size $x(t\text{-}\delta)$ we first apply the chain rule to the definition of $H(.)$ given in (3). We obtain

(A.2) $$\frac{\partial}{\partial x(t-\delta)} H(v_t^*) = -\frac{\partial F(v_t^*)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial x(t-\delta)}$$

The first term on the RHS of (A.2) is just the density of v at v_t*. To calculate the second term note that the total derivative of $u(v_t^*, x(t-\delta))$ with respect to $x(t-\delta)$ must stay constant in order to satisfy equation (A.1). This holds for

(A.3) $$\frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)} = -\frac{\partial u(v_t^*, x(t-\delta)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial x(t-\delta)}$$

Solving (A.3) for $\frac{\partial v_t^*}{\partial x(t-\delta)}$ and substituting that into (A.2) yields the result

(A.4) $$\frac{\partial}{\partial x(t-\delta)} H(v_t^*) = f(v_t^*) \cdot \left(\frac{\partial u(v_t^*, x(t-\delta)}{\partial v_t^*} \right)^{-1} \cdot \frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)},$$

where f is the density function of v.

Examination of (A.4) gives the following lemma

Lemma 1: Whenever the solution to equation (2) exists and is unique, so that v_t* is well defined, the extent of network externalities measured by $\eta \equiv \frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)}$ determines the slope of the function H in the $x(t\text{-}\delta)$ domain, such that

(i) H is non-decreasing if and only if network effects are non-negative,

(ii) the slope of H equals zero if there are no network effects, and

(iii) the slope of H increases with network effects, whenever the density of types is strictly positive.

Proof of Lemma 1: According to (A.4) the slope of function H in the $x(t\text{-}\delta)$ domain is determined by a product of the three components: The density of consumer types, the inverse of the partial derivative of the willingness-to-pay function with respect to the consumer type, and the partial derivative of the willingness-to-pay function with respect to the installed base, all evaluated at the indifferent type v_t^*. The first component of this product is non-negative (density function), the second one is positive (due to the assumed rank ordering), and the third one is the extent of network effects. ■

Analogously, to calculate the derivative of H with respect to the price p we first apply the chain rule to obtain:

(A.5) $$\frac{\partial}{\partial p} H(v_t^*) = -\frac{\partial F(v_t^*)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial p}.$$

Then we note that from (A.1) we have:

(A.6) $$\frac{\partial u(v_t^*, x(t-\delta)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial p} = 1,$$

and substitute to get:

(A.7) $$\frac{\partial}{\partial p} H(v_t^*) = -f(v_t^*) \cdot \left(\frac{\partial u(v_t^*, x(t-\delta))}{\partial v_t^*} \right)^{-1}.$$

Lemma 2 follows directly from examination of (A.7):

Lemma 2: Whenever the solution to equation (2) exists and is unique, so that v_t^* is well defined, changes in price p determine the shifts of the function H in the $x(t-\delta)$ domain, such that $H(v^*(x(t\text{-}\delta),p_1) \geq H(v^*(x(t\text{-}\delta),p_2)$ for every $x(t\text{-}\delta)$ and $H(v^*(x(t\text{-}\delta),p_1) > H(v^*(x(t\text{-}\delta),p_2)$ *for at least some* $x(t\text{-}\delta)$ if $p_1 < p_2$.

Proof of Lemma 2: Because the density of types is non-negative (and strictly positive over some range) by definition and the derivative of the willingness-to-pay function with respect to the consumer type is positive due to the assumed rank ordering, (A.7) is always non-negative and strictly positive for at least some values of the installed base $x(t-\delta)$. ■

A.2. Proofs of Propositions

Proof of Proposition 1: First, we prove that the downward-sloping parts of the long-run demand consist of stable equilibria and the upward-sloping parts are unstable. The long-run demand condition (5) implies that the long-run equlibria in our model correspond to the fixed points of the function *H(.)*. The stable fixed points of the function *H(.)* are the long-run attractors of the dynamic process described by the equation (4) (for illustration, this dynamic process is depicted by the arrows in the upper panel of Figure 1). This means that for a fixed point to be stable the function *H(.)* must cross the 45-degree line from above. The reverse is true at unstable fixed points (critical mass points): The function *H(.)* crosses the 45-degree line from below. It follows that a price decrease shifting the function *H(.)* upwards (Lemma 2), moves the stable fixed points to the right and the unstable ones to the left. Hence, downward-sloping parts of long-run demand must consist of stable equlibria and upward-sloping parts consist of unstable ones.

Second, we prove that for the unstable equilibria to exist, network effects must be strong enough. For an unstable equilibrium to exist, the function *H(.)* must cross the 45-degree line from below. This is possible if and only if the network effects are strong enough. This follows from Lemma 1, which shows the slope of function *H(.)* increases with network effects and is zero without network effects. ■

Proof of Proposition 2: The first part of Proposition 2 follows immediately from Proposition 1, which says that unstable equilibria are located on the upward-sloping part of long-run demand function. Whenever critical mass exists, an increased (decreased) price then leads to a higher (lower) critical mass. The second part of Proposition 2 follows from Lemma 1. It states that the slope of *H(.)* increases with network effects whenever the density of types is positive. Suppose that x_1 is a critical mass point, i.e. there exists a price p^* for which *H(.)* crosses the 45-degree line from below at x_1. Therefore, there must exist a neighborhood of x_1, $\left[\underline{x}_1, \overline{x}_1\right]$, such that $\underline{x}_1 < x_1 < \overline{x}_1$ and *H(.)* has a positive slope over $\left[\underline{x}_1, \bar{x}_1\right]$. Note that the density of types corresponding to $\left[\underline{x}_1, \bar{x}_1\right]$ and p^* must be strictly positive for *H(.)* to have a positive slope over $\left[\underline{x}_1, \bar{x}_1\right]$ (Lemma 1). An increase in network effects in the neighborhood $\left[\underline{x}_1, \bar{x}_1\right]$ thus increases the slope of *H(.)* over the whole neighborhood, shifting the critical mass point x_1 to the left. ■

A.3. State equations with Entry and Switching Costs

Assume a stronger version of the equal attractiveness conditions, $v_{i,t}^* = \underline{v}_t^*$ for all *i* and *t*, and substitute (18) and (21) in (22) to obtain

(A.8) $y_i(t) = H_i(v_t^*) + y_i(t-\delta) - H_i(v_{t-\delta}^*)\dfrac{\underline{I}_{t-\delta}}{\underline{I}_t}$.

One can think of (A.8) as a decomposition of the brand *i*'s installed base in time *t*. The first term on the RHS of (A.8) gives the network size of brand *i* (number of subscribers) if there were no switching costs. The second and the third term adds and subtracts, respectively, the installed base of brand *i* in a way that is sensitive to the number of active firms in the market. To see how this can lead to persistent asymmetries among firms expand the recursive equation (A.8) to

(A.9) $y_i(T) = H_i(v_T^*) + H_i(v_{T-\delta}^*) - H_i(v_{T-\delta}^*)\dfrac{\underline{I}_{T-\delta}}{\underline{I}_T} + H_i(v_{T-2\delta}^*) - H_i(v_{T-2\delta}^*)\dfrac{\underline{I}_{T-2\delta}}{\underline{I}_{T-\delta}} + \ldots + y_i(0) - H_i(v_0^*)\dfrac{\underline{I}_0}{\underline{I}_\delta}$,

where $t = 0$ indicates the time when the market starts up so there are no available networks at that time and $T > 0$.

Suppose, there is constant number of firms active in the market such that $\underline{I}_t = \underline{I}$ for $t \in (0,T)$. Then the last two terms on the RHS of (A.9) equal zero, because every firm is active from the beginning of the market, and all the middle terms cancel out. In this case (A.9) simplifies to (19), i.e. the state equations with and without switching costs are the same.

Now suppose there was an entry into the market at $t = E$, and $0 < E < T$. This means that $\underline{I}_t$ rises discontinuously in $t = E$ and stays at the higher level afterwards. The sales equations of the incumbents do not simplify to (19) any longer. Instead, they become:

$$\text{(A.10)} \quad y_i^{inc}(T) = H_i(v_T^*) + \int_E^{E+\delta} [H_i(v_{t-\delta}^*) - H_i(v_{t-\delta}^*)\frac{\underline{I}_{t-\delta}}{\underline{I}_t}]dt,$$

for $T \geq E+\delta$. The integral in (A.10) is positive. It is also invariant with respect to any events in $T > E+\delta$ and can thus be treated as a firm-specific constant in the post-entry period.

In contrast, the expansion of the recursive equation (A.8) does not go back to $t = 0$ for the entrants. Their history starts at $t = E$ and the sales can be described by

$$\text{(A.11)} \quad y_i^{ent}(T) = H_i(v_T^*) + \int_E^{E+\delta} [-H_i(v_{t-\delta}^*)\frac{\underline{I}_{t-\delta}}{\underline{I}_t}]dt,$$

for $T \geq E+\delta$. To see this, refer to (A.8) and note that $y_i^{ent}(t-\delta) = 0$ for $t \in (E, E+\delta)$. The integral in (A.11) plays analogous role for the entrants as the integral in (A.10) for incumbents, but it is negative. We can therefore conclude that incumbents have a persistent (in terms of the difference in the total sales) competitive advantage over entrants.

Moreover one can show that the fixed effects caused by entry sum up to zero. To see that, denote the number of incumbents as A and the number of entrants as B. The sum of the effects is then

$$A\int_E^{E+\delta} [H_i(v_{t-\delta}^*) - H_i(v_{t-\delta}^*)\frac{\underline{I}_{t-\delta}}{\underline{I}_t}]dt + B\int_E^{E+\delta} [-H_i(v_{t-\delta}^*)\frac{\underline{I}_{t-\delta}}{\underline{I}_t}]dt =$$

$$\text{(A.12)} \quad = A\int_E^{E+\delta} [H_i(v_{t-\delta}^*) - H_i(v_{t-\delta}^*)\frac{A}{A+B}]dt + B\int_E^{E+\delta} [-H_i(v_{t-\delta}^*)\frac{A}{A+B}]dt =$$

$$= \left(A - \frac{A^2}{A+B} - \frac{AB}{A+B}\right)\int_E^{E+\delta} H_i(v_{t-\delta}^*)\,dt = 0.$$

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Figure 1. Stable vs. unstable equilibria

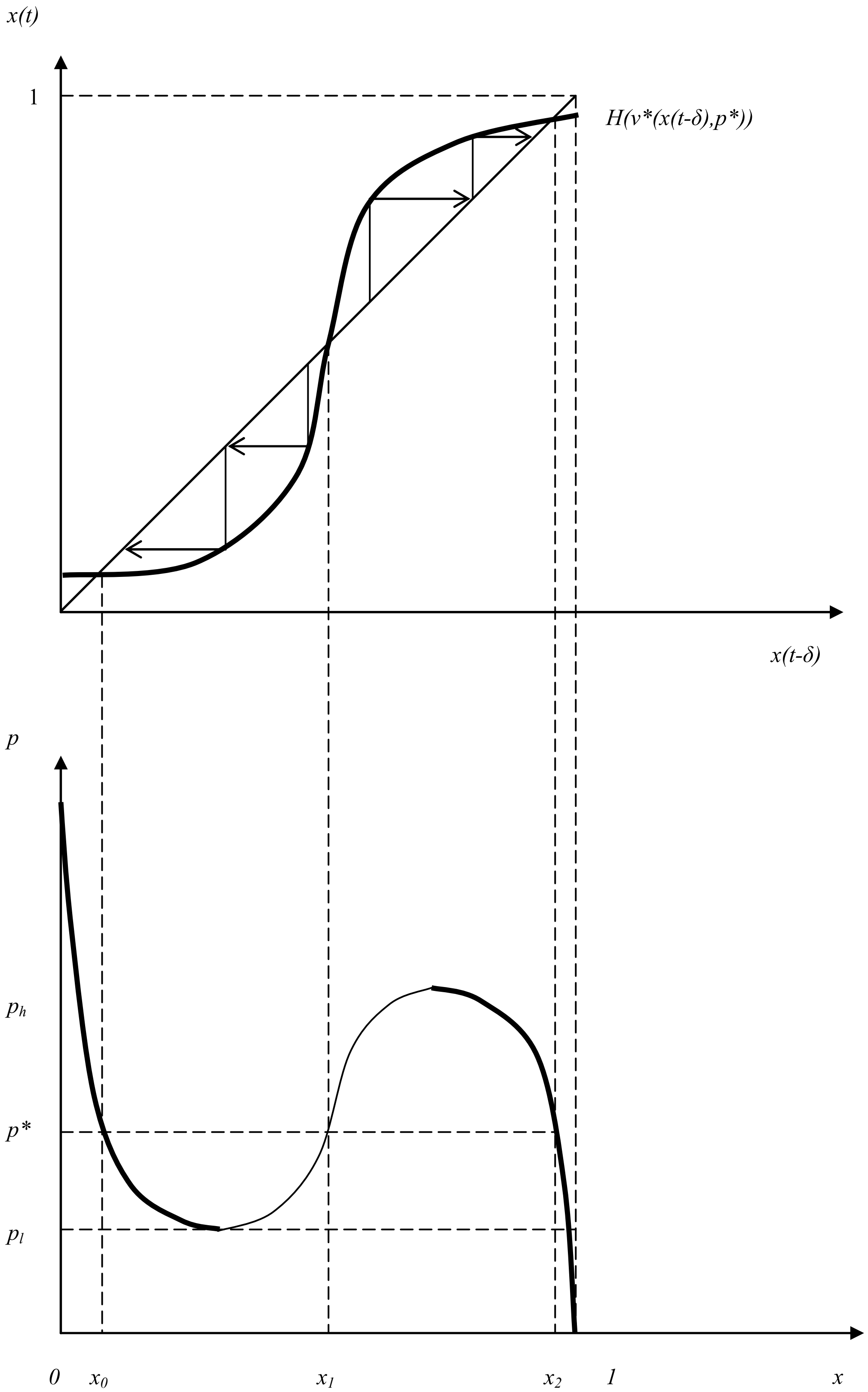


Figure 2. Simulations of the steady-state demand using different values for the network effects parameters (remaining parameters fixed: a = 100, b = 0.02).

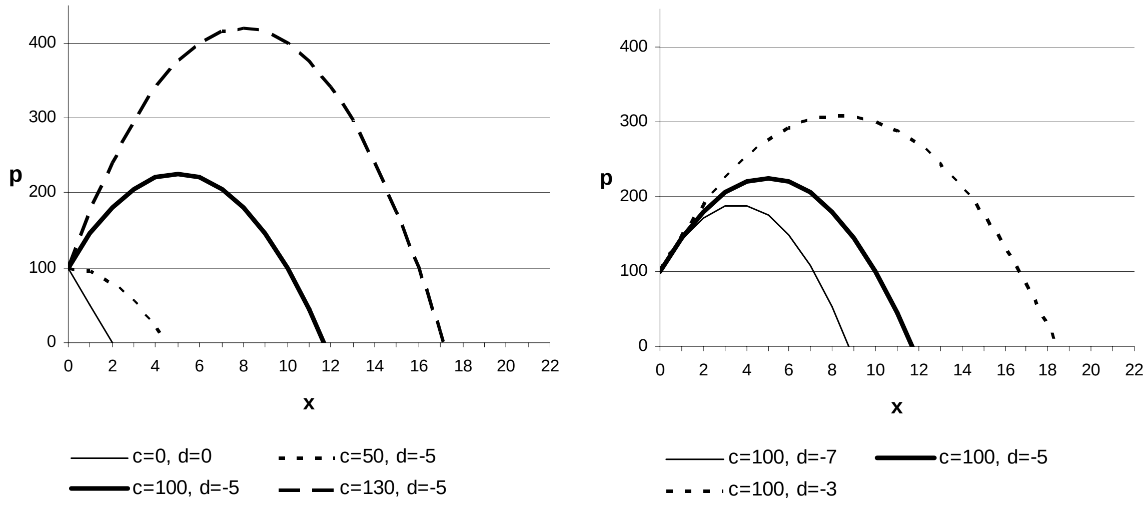
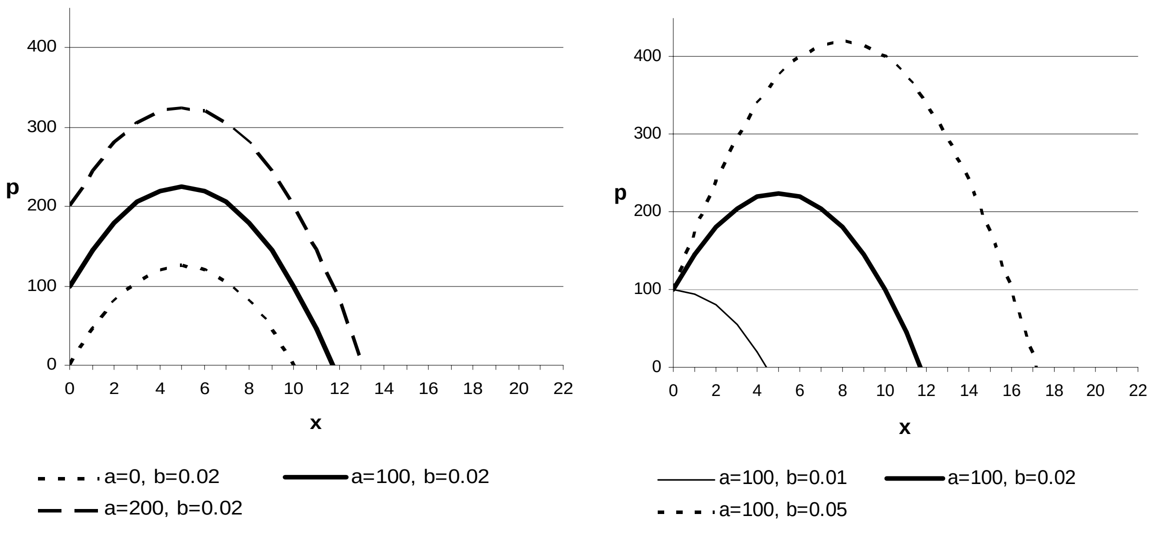


Figure 3. Simulations of the steady-state demand using different values for the parameters of consumer types' distribution (remaining parameters fixed: c = 100, d = -5).



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